Exercise 4A

1 a $\int(\sinh x+3\cosh x)\,dx=\int\sinh x\,dx+3\int\cosh x\,dx$
$=\cosh x+3\sinh x+c$

b $\int 5\operatorname{sech}^2 x\,dx=5\int\operatorname{sech}^2 x\,dx$
$=5\tanh x+c$

c $\int\frac{1}{\sinh^2 x}\,dx=\int\operatorname{cosech}^2 x\,dx$
$=-\coth x+c$

d $\int\left(\cosh x-\frac{1}{\cosh^2 x}\right)dx=\int\cosh x\,dx-\int\frac{1}{\cosh^2 x}\,dx$
$=\int\cosh x\,dx-\int\operatorname{sech}^2 x\,dx$
$=\sinh x-\tanh x+c$

e $\int\frac{\sinh x}{\cosh^2 x}\,dx=\int\frac{\tanh x}{\cosh x}\,dx$
$=\int\tanh x\operatorname{sech} x\,dx$
$=-\operatorname{sech} x+c$

f $\int\frac{3}{\sinh x\tanh x}\,dx=3\int\frac{1}{\sinh x\tanh x}\,dx$
$=3\int\operatorname{cosech} x\coth x\,dx$
$=-3\operatorname{cosech} x+c$

g $\int\operatorname{sech} x(\operatorname{sech} x+\tanh x)\,dx=\int\operatorname{sech}^2 x\,dx+\int\operatorname{sech} x\tanh x\,dx$
$=\tanh x-\operatorname{sech} x+c$

h $\int(\operatorname{sech} x+\operatorname{cosech} x)(\operatorname{sech} x+\operatorname{cosech} x)\,dx=\int(\operatorname{sech}^2 x+2\operatorname{sech} x\operatorname{cosech} x+\operatorname{cosech}^2 x)\,dx$
$=\int\left(\operatorname{sech}^2 x+\operatorname{cosech}^2 x+\frac{2}{\cosh x\sinh x}\right)dx$
$=\int\left(\operatorname{sech}^2 x+\operatorname{cosech}^2 x+\frac{2}{\frac{1}{2}\sinh 2x}\right)dx$
$=\int\left(\operatorname{sech}^2 x+\operatorname{cosech}^2 x+\frac{4}{\sinh 2x}\right)dx$
$=\int(\operatorname{sech}^2 x+\operatorname{cosech}^2 x+4\operatorname{cosech} 2x)\,dx$
$=\int\operatorname{sech}^2 x\,dx+\int\operatorname{cosech}^2 x\,dx+4\int\operatorname{cosech} 2x\,dx$
$=\tanh x-\coth x-4\coth 2x+c$

2 **a** $\int \sinh 2x \, dx = \frac{1}{2}\cosh 2x + c$

b $\int \cosh\left(\frac{x}{3}\right) dx = 3\sinh\left(\frac{x}{3}\right) + c$

c $\int \text{sech}^2(2x-1)\, dx = \frac{1}{2}\tanh(2x-1) + c$

d $\int \text{cosech}^2 5x \, dx = -\frac{1}{5}\coth 5x + c$

e $\int \text{cosech} 2x \coth 2x \, dx = -\frac{1}{2}\text{cosech} 2x + c$

f $\int \text{sech}\left(\frac{x}{\sqrt{2}}\right)\tanh\left(\frac{x}{\sqrt{2}}\right) dx = -\sqrt{2}\,\text{sech}\left(\frac{x}{\sqrt{2}}\right) + c$

g $\int \left(5\sinh 5x - 4\cosh 4x + 3\,\text{sech}^2\left(\frac{x}{2}\right)\right) dx = 5\int \sinh 5x\, dx - 4\int \cosh 4x\, dx + 3\int \text{sech}^2\left(\frac{x}{2}\right) dx$

$$= \cosh 5x - \sinh 4x + 6\tanh\left(\frac{x}{2}\right)$$

3 **a** $\int \frac{1}{1+x^2}\, dx = \arctan x + c$

b $\int \frac{1}{\sqrt{1+x^2}}\, dx = \text{arsinh} x + c$

c $\int \frac{1}{1+x}\, dx = \ln|1+x| + c$

d $\int \frac{2x}{1+x^2}\, dx = \ln|1+x^2| + c$

e $\int \frac{1}{\sqrt{1-x^2}}\, dx = \arcsin x + c$

f $\int \frac{1}{\sqrt{x^2-1}}\, dx = \text{arcosh}\, x + c$

g $\int \frac{3x}{\sqrt{x^2-1}}\, dx = 3\int (x^2-1)^{-\frac{1}{2}}\, dx$

$$= 3(x^2-1)^{\frac{1}{2}} + c$$

3 h $\int \frac{3}{(1+x)^2}\,dx = 3\int (1+x)^{-2}\,dx$

$= -3(1+x)^{-1} + c$

$= -\frac{3}{1+x} + c$

4 a $\int \frac{2x+1}{\sqrt{1-x^2}}\,dx = 2\int x(1-x^2)^{-\frac{1}{2}}\,dx + \int (1-x^2)^{-\frac{1}{2}}\,dx$

$= -2(1-x^2)^{\frac{1}{2}} + \arcsin x + c$

$= \arcsin x - 2\sqrt{1-x^2} + c$

b $\int \frac{1+x}{\sqrt{x^2-1}}\,dx = \int (x^2-1)^{-\frac{1}{2}}\,dx + \int x(x^2-1)^{-\frac{1}{2}}\,dx$

$= \text{arcosh}x + (x^2-1)^{\frac{1}{2}} + c$

$= \text{arcosh}x + \sqrt{x^2-1} + c$

c $\int \frac{x-3}{\sqrt{1+x^2}}\,dx = \int x(1+x^2)^{-\frac{1}{2}}\,dx - 3\int (1+x^2)^{-\frac{1}{2}}\,dx$

$= (1+x^2)^{\frac{1}{2}} + 3\text{arsinh}x + c$

$= \sqrt{1+x^2} + 3\text{arsinh}x + c$

5 a $\frac{x^2}{1+x^2} = \frac{1+x^2-1}{1+x^2}$

Therefore:

$\frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$ as required

b $\int \frac{x^2}{1+x^2}\,dx = \int 1\,dx - \int \frac{1}{1+x^2}\,dx$

$= x - \arctan x + c$